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# Dynamical Eightfold Way and Confinement in Strongly Coupled Lattice QCD

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## Abstract

We review our recent work validating the Eightfold Way exactly. We consider 3-flavor lattice QCD in the strong-coupling regime (small hopping parameter $\kappa > 0$ and much smaller gauge coupling $\beta > 0$) and in an imaginary-time functional integral formulation. By analyzing the subspace of the quantum mechanical Hilbert space with odd (even) number of fermions, we obtain from the dynamics the exact baryon (meson) spectrum. Using spectral representations for the 2-point functions, the hadron states are detected by isolated dispersion curves in the energy-momentum spectrum. A correlation subtraction method ensures that these hadron states are the only spectrum up to near a two-particle threshold. Hence, we do show confinement up to near the two-meson threshold. The asymptotic baryon (meson) masses are $-3\ln\kappa$ $(-2\ln\kappa)$, and are given by *convergent* expansions in $\kappa$ and $\beta$. The form of the dispersion curves is also obtained. Within the baryon octet (decuplet) all the masses are equal, but there is an $\mathcal{O}(\kappa^6)$ octet-decuplet baryon mass splitting. All mesonic masses within the pseudoscalar (vector) mesons are also the same but

there is an $\mathcal{O}(\kappa^4)$ pseudoscalar-vector meson mass splitting. A new linear time reflection symmetry is employed to derive some of the results.

## 1 Introduction and Results

In [1], a quark model with three flavors and an $\mathrm{SU}(3)_f$ flavor symmetry was introduced to classify the known hadrons by an eightfold way scheme. An $\mathrm{SU}(3)_c$ local gauge model of quarks and gluons and color dynamics was later proposed, the well-known Quantum Chromodynamics (QCD), as the model for strong interactions. Perturbation theory was successfully used for high-energy phenomena but not at low energies. To understand the low-lying energy-momentum (E-M) spectrum and confinement in QCD a lattice approximation in an imaginary-time functional integral formulation was introduced in [2]. Soon, the use of the lattice became a powerful tool in different contexts, to determine the particle content of the model and to answer questions which were not attainable using perturbation theory. For instance, several accomplishments in the strong coupling expansion are found in [3, 4, 5, 6] and numerical simulations on the lattice, which acquired a very important status, are e.g. reported in [7]. In a mathematically rigorous treatment, a physical Hilbert space $\mathcal{H}$ and E-M operators are constructed for the lattice QCD in [8]. A Feynman-Kac (F-K) formula is also established.

In a series of papers [9, 10, 11], we determined the low-lying E-M spectrum of increasingly complex $\mathrm{SU}(3)_c$ lattice QCD models in the strong coupling regime, i.e. with a hopping parameter $\kappa$ and plaquette coupling $\beta = 1/g_0^2$ satisfying $0 < \beta \ll \kappa \ll 1$. We obtained the one-hadron and the two-hadron bound-state spectra, up to a two-particle energy threshold. The reason for working in this region of parameters is that the hadron spectrum is the low-lying spectrum; baryons have asymptotic mass $\approx -3\ln\kappa$ and the meson mass is $\approx -2\ln\kappa$. If $0 < \kappa \ll \beta \ll 1$, the low-lying spectrum consists of only glueballs [12] of mass $\approx -4\ln\beta$ and their excitations.

Here, we review the results of our papers [13, 14]. We obtain the low-lying spectrum *exactly* in the $\mathrm{SU}(3)_c$ lattice QCD model with 3 flavors, in $3+1$ dimensions and in the strong coupling regime. We validate the Gell-Mann and Ne'eman eightfold way directly from the quark-gluon dynamics. Besides, we show that the spectrum associated with the eightfold way baryon (meson) states is the *only* spectrum in the subspace $\mathcal{H}_{odd}$ ($\mathcal{H}_{even}$) of the underlying Hilbert space $\mathcal{H}$ of vectors with an odd (even) number of fermions, up to the meson-baryon (meson-meson) energy threshold of $\approx -5\ln\kappa$ ($\approx -4\ln\kappa$). Since the hadronic local composite fields are $\mathrm{SU}(3)_c$ gauge invariant, we show confinement up to the meson-meson threshold. No guesswork is made in our dynamical treatment regarding the form of the hadron composite fields.

Besides the usual $\mathrm{SU}(3)_f$ quantum numbers (total hypercharge $Y$, quadratic Casimir $C_2$, total isospin $I$ and its 3rd component $I_3$), the basic excitations of our model also carry spin labels. The total spin operator $J$ and its $z$-component $J_z$ are defined using $\pi/2$ rotations about the spatial coordinate axes and agree with the infinitesimal generators of the continuum for improper zero-momentum meson states.

Regarding the baryons, we show the existence of 56 states associated with the eightfold way baryons and their anti-particles. They form the $J = 1/2$ flavor octet ($C_2 = 3$) and the $J = 3/2$ decuplet ($C_2 = 6$). Anti-baryons and baryons have the same spectral properties by charge conjugation. For the mesons, there are 36 states which can be grouped into three flavor nonets associated with the vector mesons ($J = 1$) and one nonet associated with the pseudo-scalar mesons ($J = 0$). Each nonet decomposes into an $\mathrm{SU}(3)_f$ singlet ($C_2 = 0$) and octet ($C_2 = 3$). Charge conjugation leaves invariant each of the singlets and octets. Hence, these multiplets contain their antiparticles.

All the hadrons are detected by *isolated dispersion curves* $w(\vec{p})$, $\vec{p} = (p^1, p^2, p^3) \in \mathbf{T}^3 \equiv (-\pi, \pi]^3$, in the E-M spectrum. For $\beta = 0$, we obtain

$$w(\kappa, \vec{p}) = -3\ln\kappa - 3\kappa^3/4 + \kappa^3 \sum_{j=1,2,3} (1 - \cos p^j)/4 + \kappa^6 r(\kappa, \vec{p})\,, \qquad (1)$$

for the baryons, with $r(0, \vec{p}) \neq 0$. For the mesons, we have

$$w(\vec{p}) = -2\ln\kappa - 3\kappa^2/2 + \kappa^2 \sum_{j=1,2,3} (1 - \cos p^j)/2 + \kappa^4 r(\kappa, \vec{p})\,. \qquad (2)$$

In Eq. (1), for the octet, $r(\kappa, \vec{p})$ is jointly analytic in $\kappa$ and in each $p^j$, for small $|\Im m p^j|$. A new linear symmetry called *time reflection*, in contrast with the ordinary antilinear time reversal, is used to define a spin flip symmetry in the lower (upper) indices. This symmetry is employed to show that all octet dispersion curves are identical, and the four decuplet dispersion curves are pairwise identical (depend only on $|J_z|$). The $\beta = 0$ baryon masses have all the form $M = -3\ln\kappa - 3\kappa^3/4 + \kappa^6 r(\kappa)$, with $r(\kappa)$ real analytic. We show a partial restoration of the continuous rotational symmetry at zero spatial momentum [15] which implies a same $r(\kappa)$ for all members of the octet (decuplet). So, there is no mass splitting within the octet (decuplet), but there is an octet-decuplet mass difference of $3\kappa^6/4 + \mathcal{O}(\kappa^7)$, at $\beta = 0$, which persists for $\beta \neq 0$.

In Eq. (2), $|r(\kappa, \vec{p})| \leq$ const. For the pseudo-scalar mesons $r(\kappa, \vec{p})$ is jointly analytic in $\kappa$ and $p^j$, for $|\kappa|$ and $|\Im m p^j|$ small. The meson masses are given by $m(\kappa) = -2\ln\kappa - 3\kappa^2/2 + \kappa^4 r(\kappa)$, with $r(0) \neq 0$ and $r(\kappa)$ real analytic; they are also analytic in $\beta$. For a fixed nonet, the mass of the vector mesons are independent of $J_z$ and are all equal within each octet. All singlet masses are also equal for the vector mesons. For $\beta = 0$, up to and including $\mathcal{O}(\kappa^4)$, for each nonet, the masses of the octet and the singlet are equal. All members of each octet have identical dispersions. Other dispersion curves may differ. Indeed, there is a pseudo-scalar, vector meson mass splitting (between $J = 0, 1$) given by $2\kappa^4 + \mathcal{O}(\kappa^6)$, at $\beta = 0$, which persists for $\beta \neq 0$.

## 2 Model and Spectral Analysis

Our lattice QCD model partition function is $Z = \int e^{-S(\psi,\bar{\psi},g)}\, d\psi\, d\bar{\psi}\, d\mu(g)$, and for $F(\bar{\psi}, \psi, g)$, the normalized correlations are

$$\langle\, F \,\rangle = \frac{1}{Z} \int F(\bar{\psi}, \psi, g)\, e^{-S(\psi,\bar{\psi},g)}\, d\psi\, d\bar{\psi}\, d\mu(g)\,. \tag{3}$$

The $\mathrm{SU}(3)_f$ and gauge-invariant action $S \equiv S(\psi, \bar{\psi}, g)$ is Wilson's action [3]

$$\begin{array}{rcl} S & = & \frac{\kappa}{2} \sum \bar{\psi}_{a,\alpha,f}(u) \Gamma^{\sigma e^\mu}_{\alpha\beta}\, (g_{u,u+\sigma e^\mu})_{ab} \psi_{b,\beta,f}(u + \sigma e^\mu) \\ & & + \sum_{u\in\mathbb{Z}^4_o} \bar{\psi}_{a,\alpha,f}(u) M_{\alpha\beta} \psi_{a,\beta,f}(u) - \frac{1}{g_0^2} \sum_p \chi(g_p)\,. \end{array} \tag{4}$$

Here, besides summation over $\alpha$, $\beta = 1, 2, 3, 4$ (spin), $a = 1, 2, 3$ (color) and $f = 1, 2, 3 \equiv u, d, s$ (isospin), the first sum is over $u = (u^0, \vec{u}) = (u^0, u^1, u^2, u^3) \in \mathbb{Z}^4_o \equiv \{\pm 1/2, \pm 3/2, \pm 5/2 ...\} \times \mathbb{Z}^3$, $\sigma = \pm 1$ and $\mu = 0, 1, 2, 3$. 0 is the time direction and the 3 is the $z$-direction. $e^\mu$ is the $\mu$-direction unit vector. At a site $u \in \mathbb{Z}^4_o$, $\hat{\psi}_{a\alpha f}(u)$ are Grassmann fields (the hat meaning the presence or absence of a bar). $\alpha = 1, 2$ are *upper* spin indices and $\alpha = 3, 4 \equiv +, -$ are *lower* ones. For each nearest neighbor oriented bond $< u, u \pm e^\mu >$ there is an $\mathrm{SU}(3)_c$ matrix $U(g_{u,u\pm e^\mu})$ parametrized by $g_{u,u\pm e^\mu} \in \mathrm{SU}(3)_c$, with

$U(g_{u,u+e^\mu})^{-1} = U(g_{u+e^\mu,u})$. We drop the $U$ from the notation. To each oriented plaquette $p$ there is a plaquette variable $\chi(U(g_p))$ where $U(g_p)$ is the orientation-ordered product of $\mathrm{SU}(3)_c$ matrices, and $\chi$ is $\Re e\,(\text{trace})$. $M \equiv M(m,\kappa) = 1\!\!1$, by choosing the bare quark mass $m = 1-2\kappa$. Also, we take $\Gamma^{\pm e^\mu} = -1\!\!1 \pm \gamma^\mu$, where $\gamma^0 = \begin{pmatrix} 1\!\!1 & 0 \\ 0 & -1\!\!1 \end{pmatrix}$, $\gamma^{j=1,2,3} = \begin{pmatrix} 0 & i\sigma^j \\ -i\sigma^j & 0 \end{pmatrix}$ are Dirac matrices obeying $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}1\!\!1$, where $\sigma^j$ are Pauli matrices. $d\mu(g)$ is the product measure over non-oriented bonds of normalized $\mathrm{SU}(3)_c$ Haar measures. The Grassmann integrals are Berezin integrals; for $\kappa = 0$, $\langle \psi_{\ell_1}(x)\,\bar\psi_{\ell_2}(y)\rangle = \delta_{\alpha_1,\alpha_2}\delta_{a_1a_2}\delta_{f_1f_2}\delta(x-y)$, and Wicks theorem applies.

The physical quantum mechanical Hilbert space $\mathcal{H}$ and the E-M operators $H$ and $P^j$, $j = 1,2,3$, are defined as in [8, 9]. Polymer expansion methods [8, 16] ensure the thermodynamic limit of correlations exists (below we work in this limit), and truncated correlations have exponential tree decay. The limiting correlations are lattice translational invariant and extend to analytic functions in $\kappa$ and $\beta$. For gauge-invariant $F$ and $G$ restricted to $u^0 = 1/2$, we have the F-K formula

$$(G, \check T_0^{x^0}\check T_1^{x^1}\check T_2^{x^2}\check T_3^{x^3}F)_{\mathcal{H}} = \langle [T_0^{x^0}\vec T^{\vec x}F]\Theta G\rangle \;, \tag{5}$$

where $T_0^{x^0}$, $T_i^{x^i}$, $i = 1,2,3$, denote translation of the functions of Grassmann and gauge variables by $x^0 \geq 0$, $\vec x = (x^1,x^2,x^3) \in \mathbb{Z}^3$, $T^{\vec x} = T_1^{x^1}T_2^{x^2}T_3^{x^3}$ and $\Theta$ is an antilinear, order reversing operator which involves time reflection [8]. In Eq. (5), we do not distinguish between Grassmann, gauge variables (rhs) and the associated vectors in $\mathcal{H}$ (lhs). As linear operators in $\mathcal{H}$, $\check T_{\mu=0,1,2,3}$ are mutually commuting; $\check T_0$ is self-adjoint, with $-1 \leq \check T_0 \leq 1$, and $\check T_{j=1,2,3}$ are unitary. So, $\check T_j = e^{iP^j}$ defines the self-adjoint momentum operator $\vec P = (P^1,P^2,P^3)$ with spectral points $\vec p \in \mathbf{T}^3$ and $\check T_0^2 = e^{-2H} \geq 0$ defines the energy operator $H \geq 0$. We call a point in the E-M spectrum with $\vec p = \vec 0$ a mass. Also, we let $\mathcal{E}(\lambda^0,\vec\lambda)$ be the product of the spectral families of $\check T_0$, $P^1$, $P^2$, $P^3$.

Besides the $\mathrm{SU}(3)_f$ flavor and $\mathrm{SU}(3)_c$ local gauge symmetries, the symmetries [9] of charge conjugation $\mathcal{C}$, parity $\mathcal{P}$, $\pi/2$ rotations about the spatial axes and coordinate reflections are extensively used.

Due to the lack of space, below we concentrate on $\mathcal{H}_{odd} \subset \mathcal{H}$ and the baryon spectrum. (Although more delicate[1], the analysis for the mesons in

[1]The baryon fields are $\sim \psi\psi\psi$ and are automatically truncated. The meson

$\mathcal{H}_{even} \subset \mathcal{H}$ is similar.) To show the existence of baryons up to $\approx -5\ln\kappa$, we define a matrix-valued 2-point function $G(u,v) \equiv G(u-v)$. This 2-point function has a spectral representation obtained from the F-K formula and the spectral representations of the E-M operators. Its Fourier transform $\tilde{G}(p) = \sum_{x\in\mathbb{Z}^4} G(x)e^{-ip.x}$, $p = (p^0, \vec{p})$, has a representation which allows us to relate momentum space singularities on the $\Im m p^0$ axis to points in the E-M spectrum.

It ought to be emphasized that any claimed spectral results derived from a 2-point correlation *without* a spectral representation is *not* reliable. First, a relation between this correlation and the E-M operators must be established. It is unfortunate that this basic requirement is not always satisfied! Only showing exponential decay of correlations, in principle, says nothing about the spectrum. Even when the associated decay rate is in the spectrum, we learn nothing about the spectrum *above* this point or about the nature of the spectrum. An isolated state is needed to characterize a particle.

We now sketch how our results are obtained. $\tilde{G}(p)$ has a strip of analyticity in $\Im m p^0$ which is $|\Im m p^0| \le -(3-\epsilon)\ln\kappa$, $0<\epsilon\ll 1$, leading to

$$|G(u,v)| \le \text{const}\ \kappa^{3|u-v|}\,, \tag{6}$$

with $|u-v| \equiv |u^0-v^0| + |\vec{u}-\vec{v}|$, $|\vec{u}-\vec{v}| = \sum_{i=1,2,3}|u^i-v^i|$. To show that there are isolated baryon and antibaryon dispersion curves up to $\approx -5\ln\kappa$, we consider the inverse $\tilde{\Gamma}(p) = \tilde{G}(p)^{-1}$. For fixed $\vec{p}$, $\kappa$ and $\beta$, we show that

$$\tilde{\Gamma}^{-1}(p) = \{\text{cof}\,[\tilde{\Gamma}(p)]\}^t/\text{det}\tilde{\Gamma}(p)\,, \tag{7}$$

provides a meromorphic extension of $\tilde{G}(p)$ in $p^0$. (This is a key point!) Thus, the singularities of $\tilde{G}(p)$ are zeroes of $\det\tilde{\Gamma}(p)$. The dispersions $w(\vec{p})$ verify

$$\det\tilde{\Gamma}(p^0 = iw(\vec{p}), \vec{p}, \kappa) = 0\,. \tag{8}$$

That $\tilde{\Gamma}(p)^{-1}$ does provide a meromorphic extension of $\tilde{G}(p)$ follows from the faster temporal falloff of $\Gamma(x=u-v)$, the convolution inverse of $G$. Namely,

$$|\Gamma_{\ell_1\ell_2}(u,v)| \le \text{const}\,|\kappa|^{3+5(|u^0-v^0|-1)+3|\vec{u}-\vec{v}|}\,, \tag{9}$$

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fields are $\sim \bar{\psi}\psi$ and truncation of the 2-meson function must be explicitly implemented to eliminate the vacuum state. For this, we adopt the method of duplicate field variables of [17] which complicates the analysis a bit.

for $|u^0 - v^0| \geq 1$. The faster falloff of $\Gamma(x)$ gives us analyticity in $|\Im m p^0| \leq -(5-\epsilon)\ln\kappa$, and implies the zeros of $\det\tilde{\Gamma}(p)$ are *isolated*, for each $\vec{p}$, $\kappa$ and $\beta$. To find the number and behavior of the particle dispersion curves, we need the short distance, low $\kappa$ order behavior of $\Gamma$, which follows from that of $G$.

By using the hyperplane decoupling method [9, 11], we find from the dynamics that the normalized baryon excitations fields are given by (with the simultaneous presence or absence of bars and taking *only* lower spin indices)

$$\hat{B}_{\vec{\alpha}\vec{f}}(x) = \epsilon_{abc}\,\hat{\psi}_{a\alpha_1 f_1}(x)\hat{\psi}_{b\alpha_2 f_2}(x)\hat{\psi}_{c\alpha_3 f_3}(x)/[n_{\vec{\alpha}\vec{f}}]\,. \tag{10}$$

$n_{\vec{\alpha}\vec{f}}$ is chosen such that, for coincident points, $\langle B_{\vec{\alpha}\vec{f}}\bar{B}_{\vec{\alpha}'\vec{f}'}\rangle^{(0)} = -\delta_{\vec{\alpha}\vec{\alpha}'}\delta_{\vec{f}\vec{f}'}$.

With this composite field, the 2-baryon function we use to detect baryons, for all $u$ and $v$, is given by ($\chi$ here is the Heaviside function)

$$G_{\ell_1\ell_2}(u,v) = \langle B_{\ell_1}(u)\bar{B}_{\ell_2}(v)\rangle\,\chi_{u^0\leq v^0} - \langle \bar{B}_{\ell_1}(u)B_{\ell_2}(v)\rangle^*\,\chi_{u^0>v^0}\,, \tag{11}$$

where the $\ell = (\vec{\alpha}\vec{f})$, and we suppress the lower spin indices. Letting $G = G_d + G_n$, where $G_{d,\ell_1\ell_2}(u,v) = G_{\ell_1\ell_2}(u,u)\delta_{\ell_1\ell_2}\delta_{uv}$ is the diagonal part of $G$, we define $\Gamma$ by the Neumann series $\Gamma \equiv (G_d + G_n)^{-1} = \sum_{k=0}^{\infty} G_d^{-1}(-G_n G_d^{-1})^k$, which converges by the bound of Eq. (6).

To relate points in the E-M spectrum to singularities of $\tilde{G}_{\ell_1\ell_2}(p)$ on the $\Im m p^0$ axis, we first use the F-K formula to obtain a spectral representation, with $\bar{B}_\ell \equiv \bar{B}_\ell(1/2,\vec{0})$ and $x = v - u \in \mathbb{Z}^4$, $x^0 \neq 0$,

$$\begin{array}{rcl} G_{\ell_1\ell_2}(x) & = & -(\bar{B}_{\ell_1}, \check{T}^{|x^0|}\check{T}^{\vec{x}}\bar{B}_{\ell_2})_{\mathcal{H}} \\ & = & -\int_{-1}^{1}\int_{\mathbb{T}^3}(\lambda^0)^{|x^0|-1}e^{-i\vec{\lambda}.\vec{x}}d_\lambda(\bar{B}_{\ell_1}, \mathcal{E}(\lambda^0,\vec{\lambda})\bar{B}_{\ell_2})_{\mathcal{H}}\,, \end{array} \tag{12}$$

which is an even function of $\vec{x}$ by the $\mathcal{P}$ symmetry. For the Fourier transform, after separating out the $x^0 = 0$ contribution, we get

$$\tilde{G}_{\ell_1\ell_2}(p) \;=\; \tilde{G}_{\ell_1\ell_2}(\vec{p}) - (2\pi)^3\int_{-1}^{1} f(p^0,\lambda^0)d_{\lambda^0}\alpha_{\vec{p},\ell_1\ell_2}(\lambda^0)\,, \tag{13}$$

with $f(x,y) \equiv (e^{ix}-y)^{-1} + (e^{-ix}-y)^{-1}$, where $d_{\lambda^0}\alpha_{\vec{p},\ell_1\ell_2}(\lambda^0) = \int_{\mathbb{T}^3}\delta(\vec{p}-\vec{\lambda})\, d_{\lambda^0}d_{\vec{\lambda}}(\bar{B}_{\ell_1}, \mathcal{E}(\lambda^0,\vec{\lambda})\bar{B}_{\ell_2})_{\mathcal{H}}$, and we have set $\tilde{G}(\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}}G(x^0=0,\vec{x})$.

As seen from Eq. (13), singularities on the $\Im m\, p^0$ axis are spectral points and are contained in the zeroes of $\det\tilde{\Gamma}(p)$.

We consider first the determination of the baryon masses (i.e. $\vec{p} = \vec{0}$). We pass to a basis where $\tilde{\Gamma}(p^0, \vec{p} = \vec{0})$ is diagonal. The diagonalization is achieved by exploiting the $\mathrm{SU}(3)_f$ symmetry, and passing to the eightfold way baryon particle basis. The new basis and the individual spin and isospin basis we have used hitherto are related by an orthogonal transformation. The octet and the decuplet basis are the usual ones from the continuum [1]. Applying the $\mathrm{SU}(3)_f$ symmetry reduces $\tilde{\Gamma}(p)$ to a block form with 8 identical $2\times 2$ blocks associated with the octet, and 10 identical $4\times 4$ blocks associated with the decuplet. Using $\pi/2$ rotations about $e^3$, $\tilde{\Gamma}(p)$ is diagonal at $\vec{p}=\vec{0}$; and by $e^1$ reflections the elements only depend on $|J_z|$. However, the partial restoration of continuous rotational symmetry at zero spatial momentum [15] shows that the masses are independent of $J_z$. The determinant factorizes, and we consider one of the 56 typical factors (for which we omit all indices).

Next, we employ the auxiliary function method [16] to determine convergent expansions for the baryon octet and the decuplet masses, and the $\mathcal{O}(\kappa^6)$ octet-decuplet mass splitting. Our method works for all $\beta \ll \kappa \ll 1$, but here we analyze only the leading $\beta=0$ case, for simplicity. As the mass $\nearrow \infty$ as $\kappa \searrow 0$, the usual implicit function theorem does not apply to solve Eq. (8) at $\vec{p}=\vec{0}$. We make a nonlinear transformation from $p^0$ to $w=-1-c_3(\vec{p})\kappa^3+\kappa^3 e^{-ip^0}$, with $c_3(\vec{p})=-\sum_{j=1,2,3}\cos p^j/4$, and introduce an auxiliary function $H(w,\kappa)$ such that $\tilde{\Gamma}(p^0,\vec{p})=H(w=-1-c_3(\vec{p})\kappa^3+\kappa^3 e^{-ip^0},\kappa)$. With this, the non-singular part of the mass $M+3\ln\kappa$ is brought from infinity to close to $w=0$, as $\kappa\searrow 0$. Using $\mathcal{T}$ and $\mathcal{P}$ $[\Gamma(x^0,\vec{x})=\Gamma(-x^0,\vec{x})]$, we have

$$\begin{aligned} H(w,\kappa) \;=\;& \textstyle\sum_{\vec{x}}\Gamma(x^0=0,\vec{x})e^{-i\vec{p}.\vec{x}}+\sum_{\vec{x},n=1,2\ldots}\Gamma(n,\vec{x}) \\ &\times\left[\left(\tfrac{1+w+c_3(\vec{p})\kappa^3}{\kappa^3}\right)^n+\left(\tfrac{\kappa^3}{1+w+c_3(\vec{p})\kappa^3}\right)^n\right]e^{-i\vec{p}.\vec{x}}. \end{aligned} \tag{14}$$

The bound on $\Gamma$ of Eq. (9) guarantees that $H(w,\kappa)$ is jointly analytic in $\kappa$ and $w$, $|\kappa|$, $|w|\ll 1$. To control the mass to $\mathcal{O}(\kappa^6)$, we need the low $\kappa$ order short distance behavior of $\Gamma(x)$, which follows from that of $G(x)$. Precisely, we need $\Gamma(x^0=n,\vec{x})/\kappa^{3n})$ up to and including $\mathcal{O}(\kappa^6)$. At $\kappa=0$, $G(x=0)=-1$ implies $\Gamma(x=0)=-1$, and $G(x=e^0)=-\kappa^3+\mathcal{O}(\kappa^4)$ implies $\Gamma(x=e^0)=\kappa^3+\mathcal{O}(\kappa^4)$. Other contributions follow from the coefficients of the $\kappa$ expansion of $G$. Namely, there are contributions arising from non-intersecting paths connecting the point 0 to $x$ and paths that emit and absorb a meson. Using these short-distance results, after a lengthy calculation, we find

$$H(w,\kappa)=w+\frac{\kappa^6}{1+w}+a_6\kappa^6+b\kappa^6+\kappa^6\sum_{n=1,\ldots,4}c'_{3n+6}(1+w)^n+h(w,\kappa)\kappa^7\,, \tag{15}$$

with the same $b$ and $c'$s for the octet and the decuplet, and $h(w,\kappa)$ jointly analytic in $w$ and $\kappa$. The term $a_6\kappa^6$ comes from $x = \epsilon e^i + \epsilon' e^j$, $ij = 12, 13, 23$, $\epsilon, \epsilon' = \pm 1$, which we call *spatial angles*. $a_6$ is equal to $a_o = 3/8$ ($a_d = -3/8$) for the octet (decuplet) and gives the mass splitting $M_d - M_o = 3\kappa^6/4 + \mathcal{O}(\kappa^7)$. As $H(0,0) = 0$ and $[\partial H/\partial w](0,0) = 1$, the analytic implicit function theorem implies that $H(w,\kappa) = 0$ has the analytic solution $w(\kappa) = -a_6\kappa^6 - b'\kappa^6 + \mathcal{O}(\kappa^7)$, with $b' = b + 1 + \sum_{n=1,\ldots,4} c'_{3n+6}$.

For $\beta \neq 0$, the arguments above hold since $H(w,\kappa,\beta)$ is jointly analytic in $w$, $\kappa$, and $\beta$ and $[\partial H/\partial w](w=0,\kappa=0,\beta=0) = 1 \neq 0$. $w(\kappa,\beta)$ is jointly analytic in $\kappa$ and $\beta$. The non-singular contribution to the mass is also jointly analytic in $\kappa$ and $\beta$. In particular, the mass splitting persists for $0 < \beta \ll \kappa$.

For the $\vec{p} \neq \vec{0}$ dispersion curves, the $2 \times 2$ and the $4 \times 4$ blocks of $\tilde{\Gamma}(p)$ still have a complicated structure even after the use of the usual well known symmetries. However, we have found a new local symmetry of spin flip $\mathcal{F}_s$, which is a composition of $\mathcal{T}$, $\mathcal{C}$ and a nonlocal, linear time reflection, and which we use to simplify $\tilde{\Gamma}(p)$. The action of $\mathcal{F}_s$ on single Fermi fields is such that $\hat{\psi}_1 \to \hat{\psi}_2$, $\hat{\psi}_2 \to -\hat{\psi}_1$, $\hat{\psi}_3 \to \hat{\psi}_4$ and $\hat{\psi}_4 \to -\hat{\psi}_3$. For functions of the gauge fields, $f(g_{xy}) \to \bar{f}(g^*_{xy})$. With this, the action of Eq. (4) is *termwise invariant* and we have a symmetry of the system satisfying $\langle F \rangle = \langle \mathcal{F}_s F \rangle^*$.

For the $2 \times 2$ block of $G$ of the octet, using $\mathcal{F}_s$ shows that the blocks are diagonal and a multiple of the identity. The identical dispersion curves $w(\vec{p})$ can be obtained by using the auxiliary function method as before. For the $4 \times 4$ decuplet blocks, $\mathcal{F}_s$ simplifies the matrix $\tilde{\Gamma}(\vec{p})$ but it is *not* diagonal. We have not been able to apply the auxiliary function method. However, we can use a Rouché theorem [16] argument (principle of the argument), to show that there are exactly four pairwise identical solutions for each fixed $\vec{p}$.

The above analysis shows the existence of baryons in the subspace of $\mathcal{H}$ generated by the baryon fields. To extend the results to the whole $\mathcal{H}_{odd}$, up to near the meson-baryon threshold of $\approx -5 \ln \kappa$, we use $G(x)$ to define a generalized subtracted 2-point function $\mathcal{G}(x)$ and show that $\tilde{\mathcal{G}}(p)$ is regular in $|\Im m p^0| \leq -(5-\epsilon)\ln \kappa$, showing an upper mass gap property.

## 3 Conclusions

We have validated the eightfold way in strongly coupled lattice QCD with 3 flavors, by showing the energy-momentum spectrum consists exactly of the eightfold way hadrons up to near the meson-meson threshold, showing confinement up to near this threshold. The determination of the one-hadron spectrum is a necessary step to analyze the existence of bound states. With our method we can access the hadron-hadron spectrum, and should be able to help in clarifying fundamental open questions as e.g. the existence of certain tetraquark and pentaquark states as e.g. meson-meson and meson-baryon bound-states.

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