On Baryon-Baryon Bound States in a $(2+1)$-dimensional Lattice QCD Model

Paulo A. Faria da Veiga* and Michael O'Carroll†
Departamento de Matemática, ICMC-USP, C.P. 668, 13560-970 São Carlos, SP, Brazil.

Ricardo Schor‡
Departamento de Física-ICEx, UFMG, C.P. 702, 30161-970 Belo Horizonte MG, Brazil
(Dated: May 14, 2003)

We consider bound states of two baryons (anti-baryons) in lattice QCD in an Euclidean formulation. For simplicity, we analyze an $SU(3)$ theory with a single flavor in $2+1$ dimensions and two-dimensional Dirac matrices. For small hopping parameter $0 < \kappa << 1$ and large glueball mass, recently we showed the existence of a (anti-)baryon-like particle, with asymptotic mass of order $-3\ln\kappa$ and with an isolated dispersion curve, i.e. an upper gap property persisting up to near the meson-baryon threshold which is of order $-5\ln\kappa$. Here, we show there is no baryon-baryon (or anti-baryon, anti-baryon) bound state solution to the Bethe-Salpeter equation up to the two-baryon threshold which is approximately $-6\ln\kappa$.

PACS numbers: 12.38.Gc, 11.10.St, 11.15.Ha, 24.85.+p

The problem of the existence of QCD as well as its particle spectrum and scattering content are fundamental problems in physics. Given the existence of baryons there is also the problem of understanding the forces between baryons (i.e. nuclear forces) from first principles, and thus bridging the gap between QCD and the effective baryon, effective meson picture of nuclear forces through single and multiple boson exchange arising from a Yukawa interaction (see [1, 2]). In the lattice approximation to QCD much progress has been made towards understanding the low-lying energy-momentum (e-m) spectrum both at the theoretical and numerical level (see [3–11]). For small hopping parameter $0 < \kappa << 1$ and large glueball mass, here we consider the existence of baryon-baryon bound states in the context of lattice QCD in the Euclidean formulation (see [5, 6]). For simplicity, we consider an $SU(3)$ theory with a single flavor in $2+1$ dimensions and two-dimensional Dirac matrices. Recently, in [12] we showed the existence of a baryon (anti-baryon)-like particle with an isolated dispersion curve, ensuring the upper gap property, in the full baryon sector, up to near the meson-baryon threshold $-5\ln\kappa$. (An upper gap property has not been established in the Hamiltonian formulation treatments.) The corresponding (anti-)baryon mass is asymptotically of order $-3\ln\kappa$. Analogous to the Källen-Lehman representation in quantum field theory, a spectral representation was also obtained in [12] for the two-particle correlation function which allows us to relate the singularities of its Fourier transform to the energy-momentum spectrum via a Feynman-Kac formula. Furthermore, the symmetries of the model were established and analyzed. In this report, we show that there are no baryon-baryon (or anti-baryon-anti-baryon) bound states, up to the two-baryon threshold which is approximately $-6\ln\kappa$, using the ladder approximation to the Bethe-Salpeter (B-S) equation. However, adapting the methods of [13], it can be shown that the bound state non-existence result extends beyond the ladder approximation to the full model. Whether or not the absence of bound states here is a consequence of using 2×2 rather than 4×4 Dirac matrices, one rather than two or more flavors, and even a large glueball mass and/or small hopping parameter is to be investigated. In this way, in [14] we have established the existence of baryon particles in the more realistic lattice model in $d = 2, 3$ space dimensions and 4×4 Dirac matrices. Mass splitting and symmetries were also determined. Similar results are given for the meson sector in [15] (see also [7]).

Here, we consider the same SU(3) QCD lattice model as in [12], in $d = 2$ space dimensions, with 2×2 Dirac spin matrices and one flavor. The partition function is given formally by

$$Z = \int e^{-S(\psi,\bar\psi,g)}\, d\psi\, d\bar\psi\, d\mu(g)\,, \tag{1}$$

where the model action $S(\psi,\bar\psi,g)$ is

$$S(\psi,\bar\psi,g) = \tfrac{\kappa}{2}\sum \bar\psi_{\alpha,a}(u)\Gamma^{\epsilon e^\mu}_{\alpha\beta}(g_{u,u+\epsilon e^\mu})_{ab}\psi_{\beta,b}(u+\epsilon e^\mu) + \sum_{u\in\mathbb{Z}^3_o}\bar\psi_{\alpha,a}(u)M_{\alpha\beta}\psi_{\beta,a}(u) - \tfrac{1}{g_0^2}\sum_p \chi(g_p)\,, \tag{2}$$

where the first sum runs over $u\in\mathbb{Z}^3_o$, $\epsilon = \pm 1$ and $\mu = 0,1,2$. For $F(\bar\psi,\psi,g)$, the normalized expectations are denoted by $\langle F\rangle$.

We use the same notation and convention as appears in [12] and for the treatment of symmetries we refer to [14, 15]. Here, we recall that the Fermi fields $\psi_{\alpha,a}(u)$, $a = 1,2,3$, $\alpha = 1,2 \equiv +,-$, $u = (u^0,\vec u) = (u^0,u^1,u^2)$, are defined on the lattice with half-integer time coordinates $u\in\mathbb{Z}^3_o \equiv \mathbb{Z}_{1/2}\times\mathbb{Z}^2$, where $\mathbb{Z}_{1/2} = \{\pm 1/2, \pm 3/2, \ldots\}$. Letting e^μ, $\mu = 0,1,2$, denote the unit lattice vectors, there is a gauge group matrix $U(g_{u+e^\mu,u}) = U(g_{u,u+e^\mu})^{-1}$

*Electronic address: veiga@icmc.usp.br
†Electronic address: ocarroll@icmc.usp.br
‡Electronic address: rsschor@fisica.ufmg.br

associated with the directed bond $u, u+e^{\mu}$, and we drop U from the notation. We assume the hopping parameter κ to be small and much larger than the plaquette coupling parameter $g_0^{-2} > 0$. Without loss of generality, the parameter $m > 0$ is fixed such that $M = m + 2\kappa = 1$. By polymer expansion methods (see [6, 16]), the thermodynamic limit of correlations exists and truncated correlations have exponential tree decay. The limiting correlation functions are lattice translational invariant. Furthermore, the correlation functions extend to analytic functions in the coupling parameters.

The physical quantum mechanical Hilbert space $\mathcal{H}$ and the e-m operators are obtained starting from gauge invariant correlation functions, with support restricted to $u^0 = 1/2$, by a standard construction. Letting $T_0^{x^0}$, $T_i^{x^i}$, $i = 1, 2$, denote translation of the functions of Grassmann and gauge variables by $x^0 \geq 0$, $x \in \mathbb{Z}^3$; and for F and G only depending on coordinates with $u^0 = 1/2$, we have the Feynman-Kac (F-K) formula

$$(G, T_0^{x^0} T_1^{x^1} T_2^{x^2} F)_{\mathcal{H}} = \langle [T_0^{x^0} T_1^{x^1} T_2^{x^2} F] \Theta G \rangle \ ,$$

where Θ is an anti-linear operator which involves time reflection. We do not distinguish between Grassmann, gauge variables and their associated Hilbert space vectors in our notation. As linear operators in $\mathcal{H}$, T_μ, $\mu = 0, 1, 2$, are mutually commuting; T_0 is self-adjoint, with $-1 \leq T_0 \leq 1$, and $T_{j=1,2}$ are unitary, so that we write $T_j = e^{iP^j}$ and $\vec{P} = (P^1, P^2)$ is the self-adjoint momentum operator, with spectral points $\vec{p} \in \mathbf{T}^2 \equiv (-\pi, \pi]^2$. Since $T_0^2 \geq 0$, we define the energy operator $H \geq 0$ by $T_0^2 = e^{-2H}$, and refer to each point in the e-m spectrum associated with zero-momentum as mass.

We work in the subspace $\mathcal{H}_e \subset \mathcal{H}$ generated by an even number of $\hat{\psi} = \bar{\psi}$ or ψ. For the pure gauge case and small g_0^{-2}, the low-lying glueball spectrum is found in [17]. For large g_0, the glueball mass is $\approx 8 \ln g_0$.

We now turn to our results and show how they are obtained. Recently, considering the odd subspace $\mathcal{H}_o \subset \mathcal{H}$ and for small enough $\kappa >> g_0^{-2}$, and for energies less than $-(5-\epsilon)\ln\kappa$, $0 < \epsilon << 1$, in [12] we showed that in $\mathcal{H}_o$ that the low-lying e-m spectrum consists of a three-quark baryon-like particle and its anti-particle. The baryon state is associated with $\bar{\phi}_-(u)$ and the anti-baryon with $\phi_+(u)$, where $\phi_\pm(u)$ are vectors associated with Grassmann gauge invariant baryon fields given by

$$\hat{\phi}_\pm \left(u^0 = 1/2, \vec{u}\right) = \frac{1}{6} \epsilon_{a,b,c} \hat{\psi}_{a\pm} \hat{\psi}_{b\pm} \hat{\psi}_{c\pm} = \hat{\psi}_{1\pm} \hat{\psi}_{2\pm} \hat{\psi}_{3\pm} \ ,$$

with either three ψ's or three $\bar{\psi}$'s. The associated two-point baryon function is $G_-(u,v) = \langle \phi_-(u) \bar{\phi}_-(v) \rangle \, \chi_{u^0 \leq v^0} - \langle \bar{\phi}_-(u) \phi_-(v) \rangle^* \, \chi_{u^0 > v^0} = G_-(u - v)$, where χ here denotes the characteristic function. $G_-(x)$, $x^0 \neq 0$, admits the spectral representation

$$G_-(x) = -\int_{-1}^{1} \int_{\mathbf{T}^2} (\lambda^0)^{|x^0|-1} e^{i\vec{\lambda}.\vec{x}} d_{\lambda^0} \alpha_{\vec{\lambda}}(\lambda^0) d\vec{\lambda} \,,$$

where $d_{\lambda^0} \alpha_{\vec{\lambda}}(\lambda^0) d\vec{\lambda} = d_{\lambda^0} d_{\vec{\lambda}} (\bar{\phi}_-, \mathcal{E}(\lambda^0, \vec{\lambda}) \bar{\phi})_{\mathcal{H}}$ and $\mathcal{E}$ is the product of the spectral families for the energy and momentum component operators. For its Fourier transform $\tilde{G}_-(p) = \sum_{x \in \mathbb{Z}^3} e^{-ip.x} G_-(x)$, $p = (p^0, \vec{p}) \in \mathbf{T}^3$, we get

$$\begin{aligned} \tilde{G}_-(p) \ &= \ \tilde{G}_-(\vec{p}) - (2\pi)^2 \textstyle\int_{-1}^{1} \int_{\mathbf{T}^2} f(p^0, \lambda^0) \\ &\quad \times \ \delta(\vec{p} - \vec{\lambda}) d_{\lambda^0} \alpha_{\vec{\lambda}}(\lambda^0) d\vec{\lambda} \,, \end{aligned}$$

where $f(x,y) \equiv (e^{ix} - y)^{-1} + (e^{-ix} - y)^{-1}$, $\tilde{G}_-(\vec{p}) = \sum_{x \in \mathbb{Z}^2} e^{-i\vec{p}.\vec{x}} G_-(x^0 = 0, \vec{x})$, $\bar{\phi}_- = \bar{\phi}_-(1/2, \vec{0})$. There is a similar definition for the associated two-point anti-baryon function $G_+(u,v)$. From the symmetry results from [12, 14], charge conjugation shows that all baryon and anti-baryon spectral properties are the same. Thus, from now on, we restrict our attention to G_- and, whenever there is no confusion, suppress the subscript from the notation. The associated dispersion relation is

$$\begin{aligned} w(\vec{p}) \ &= \ -3\ln\kappa + r(\kappa, \vec{p}) \\ &= \ 3\ln\tfrac{M}{2\kappa} + \ln[1 - 2\tfrac{\kappa^3}{M^3}(\cos p^1 + \cos p^2)] + \mathcal{O}(\kappa^4) \,, \end{aligned}$$

with $r(\kappa, \vec{p})$ real analytic in κ and each component p^j ($j = 1, 2$). Clearly, $w(\vec{p}) \approx m_\kappa + \frac{\kappa^3}{M^3} |\vec{p}\,|^2$, $|\vec{p}| \ll 1$, where $m_\kappa \equiv w(\vec{0})$ is the (anti-) baryon mass. Furthermore, the spectral measure has the decomposition $d_{\lambda^0} \alpha_{\vec{\lambda}}(\lambda^0) = Z(\vec{p}) \delta(\lambda^0 - e^{-w(\vec{\lambda})}) d\lambda^0 + d\nu(\lambda^0, \vec{\lambda})$, where, for $\tilde{\Gamma}(p) = \tilde{G}(p)^{-1}$, we have $Z(\vec{p})^{-1} = -(2\pi)^2 \ e^{w(\vec{p})} \frac{\partial \tilde{\Gamma}}{\partial \chi} (p^0 = i\chi, \vec{p})|_{\chi = w(\vec{p})}$, such that $Z(\vec{p}) = -(2\pi)^{-2} e^{-w(\vec{p})} + \mathcal{O}(\kappa^4)$, with $Z(\vec{p})$ also real analytic in κ and p^j, $j = 1, 2$. The λ^0 support of $d\nu(\lambda^0, \vec{\lambda})$ is contained in $|\lambda^0| \leq |\kappa|^{5-\epsilon}$ and $\int_{-1}^{1} d\nu(\lambda^0, \vec{\lambda}) \leq \mathcal{O}(\kappa^4)$. Points in the spectrum occur as p^0 singularities of $\tilde{G}(p)$, for fixed $\vec{p}$, and the baryon mass points occur as singularities for $p^0 = \pm i w(\vec{p})$. Our analysis shows that points of the form $p^0 = \pi + i\chi$, $|\chi| < -(5-\epsilon)\ln\kappa$, are regular. Notice that the above measure decomposition shows the dispersion curve is isolated up to $-(5-\epsilon)\ln\kappa$ (upper gap property), making possible the particle identification.

To determine the existence of $\bar{\phi}_- \bar{\phi}_-$ bound states, we consider the associated four-point functions (with $u_1^0 = u_2^0 = u^0$ and $u_3^0 = u_4^0 = v^0$)

$$\begin{aligned} &\mathcal{G}_-(u_1, u_2, u_3, u_4) = \langle \phi_-(u_1) \phi_-(u_2) \bar{\phi}_-(u_3) \bar{\phi}_-(u_4) \rangle \\ &\times \ \chi_{u^0 \leq v^0} + \langle \bar{\phi}_-(u_1) \bar{\phi}_-(u_2) \phi_-(u_3) \phi_-(u_4) \rangle^* \chi_{u^0 > v^0} \,. \end{aligned}$$

Similarly, we define $\mathcal{G}_+(u_1, u_2, u_3, u_4)$ associated with $\phi_+ \phi_+$ bound states; which equals $\mathcal{G}_-$ by charge conjugation. We then restrict our attention to $\mathcal{G}_- \equiv \mathcal{G}$ and give a rough description of our method before going into detail. Similar methods have been employed in [13] to determine bound state spectrum of the transfer matrix associated with classical ferromagnetic spin systems at high temperature. We first obtain a spectral representation for $\mathcal{G}(x)$, where $\mathcal{G}(x) = \mathcal{G}(u_1, u_2, u_3 + \vec{x}, u_4 + \vec{x})$, with $x = (x^0 = v^0 - u^0, \vec{x}) \in \mathbb{Z}^3$, and its Fourier transform $\tilde{\mathcal{G}}(k)$. In this way, we can relate k singularities in $\tilde{\mathcal{G}}(k)$ to the e-m spectrum. Next, using a lattice B-S equation in the ladder approximation, we look for the singularities of $\tilde{\mathcal{G}}(p)$ below the two baryon threshold.

From the Feynman-Kac formula, for $x^0 \neq 0$, we have $\mathcal{G}(x) = -(\bar\phi_-(1/2,\vec u_1)\bar\phi_-(1/2,\vec u_2),(T^0)^{|x^0|-1}\check{T}^{\vec x} \times \bar\phi_-(1/2,\vec u_3)\,\bar\phi_-(1/2,\vec u_4))_{\mathcal{H}}$. With the same notation as before, upon taking the Fourier transform and inserting the spectral representations for T^0, T^1 and T^2, we have

$$\begin{aligned}\tilde{\mathcal{G}}(k) = \tilde{\mathcal{G}}(\vec k) + (2\pi)^2 \textstyle\int_{-1}^{1}\int_{\mathbf{T}^2}\, f(k^0,\lambda^0)\,\delta(\vec k-\vec\lambda)\, \times \\ d_\lambda d_{\vec\lambda}(\bar\phi_-(1/2,\vec u_1)\bar\phi_-(1/2,\vec u_2), \\ \times\ \mathcal{E}(\lambda^0,\vec\lambda)\bar\phi_-(1/2,\vec u_3)\bar\phi_-(1/2,\vec u_4))_{\mathcal{H}}\,.\end{aligned}$$

where $\tilde{\mathcal{G}}(\vec k) = \sum_{\vec x\in\mathbf{T}^2} e^{-i\vec k.\vec x}\mathcal{G}(x^0=0,\vec x)$. The singularities in $\tilde{\mathcal{G}}(k)$, for $k=(k^0=i\chi,\vec k=0)$ and $e^{\pm\chi}\le 1$, are points in the mass spectrum, i.e. the e-m spectrum at system momentum zero.

To analyze $\tilde{\mathcal{G}}(k)$, we follow the analysis for spin models as in [13]. We relabel the time direction coordinates in $\mathcal{G}(x)$ by integer labels, with $u_i^0 - 1/2 = x_i^0$, $\vec u_i = \vec x_i$, $i=1,\dots 4$, and write $D(x_1,x_2,x_3+\vec x,x_4+\vec x)$, $x_1^0 = x_2^0$ and $x_3^0 = x_4^0$, $x^0 = x_3^0 - x_2^0$, where x_i and x are points on the $\mathbb{Z}^3$ lattice. Now we pass to difference coordinates and then to lattice relative coordinates $\xi = x_2 - x_1$, $\eta = x_4 - x_3$ and $\tau = x_3 - x_2$ to obtain $D(x_1,x_2,x_3+\vec x,x_4+\vec x) = D(0,x_2-x_1,x_3-x_1+\vec x,x_4-x_1+\vec x) \equiv D(\vec\xi,\vec\eta,\tau+\vec x)$ and $\tilde{\mathcal{G}}(k) = e^{i\vec k.\vec\tau}\hat D(\vec\xi,\vec\eta,k)$, where $\hat D(\vec\xi,\vec\eta,k) = \sum_{\tau\in\mathbb{Z}^3} D(\vec\xi,\vec\eta,\tau)e^{-ik.\tau}$. Explicitly, we have

$$\begin{aligned}D(x_1,x_2,x_3,x_4) = \langle\phi_-(x_1^0+1/2,\vec x_1)\phi_-(x_2^0+1/2,\vec x_2) \\ \bar\phi_-(x_3^0+1/2,\vec x_3)\bar\phi_-(x_4^0+1/2,\vec x_4)\rangle\,\chi_{x_2^0\le x_3^0} \\ +\ \langle\bar\phi_-(x_1^0+1/2,\vec x_1)\bar\phi_-(x_2^0+1/2,\vec x_2) \\ \phi_-(x_3^0+1/2,\vec x_3)\phi_-(x_4^0+1/2,\vec x_4)\rangle^*\,\chi_{x_2^0> x_3^0}\,.\end{aligned}$$

The point of all this is that the singularities of $\tilde{\mathcal{G}}(k)$ are the same as those of $\hat D(\vec\xi,\vec\eta,k)$ and the B-S equation for $\hat D(\vec\xi,\vec\eta,k)$ and its analysis are familiar and have been treated before in [13]. The difference from the boson case treated in [13] is that here we have anti-symmetry in the $x_1 \leftrightarrow x_2$ and $x_3 \leftrightarrow x_4$ interchanges. The B-S equation in operator form, and in what we call the equal time representation, is $D = D_0 + D_0KD$. In terms of kernels,

$$\begin{aligned}D(x_1,x_2,x_3,x_4) = D_0(x_1,x_2,x_3,x_4)\, + \\ \textstyle\int D_0(x_1,x_2,y_1,y_2)K(y_1,y_2,y_3,y_4)D(y_3,y_4,x_3,x_4)\,\times \\ \delta(y_1^0-y_2^0)\delta(y_3^0-y_4^0)\,dy_1dy_2dy_3dy_4\,;\ x_1^0=x_2^0\,,\ x_3^0=x_4^0\,,\end{aligned}$$

where $D_0(x_1,x_2,x_3,x_4) = -G(x_1,x_3)G(x_2,x_4) + G(x_1,x_4)G(x_2,x_3)$, and we use a continuum notation for sums over lattice points. D, D_0 and $K = D_0^{-1} - D^{-1}$ are to be taken as matrix operators acting on $\ell_2^a(\mathcal{A})$, the anti-symmetric subspace of $\ell_2(\mathcal{A})$, where $\mathcal{A} = \left\{(x_1,x_2)\in\mathbb{Z}^3\times\mathbb{Z}^3/x_1^0 = x_2^0\right\}$. In terms of the $(\vec\xi,\vec\eta,\tau)$ relative coordinates and taking the Fourier transform in τ only, the B-S equation becomes (see [13])

$$\begin{aligned}\hat D(\vec\xi,\vec\eta,k) = \hat D_0(\vec\xi,\vec\eta,k) \\ +\textstyle\int\ \hat D_0(\vec\xi,\vec\xi\,',k)\hat K(-\vec\xi\,',-\vec\eta\,',k)\hat D(\vec\eta\,',\vec\eta,k)\,d\vec\xi\,'d\vec\eta\,'\,.\end{aligned}$$

With k fixed, $\hat D(\vec\xi,\vec\eta,k)$, etc, are taken as a matrix operator on $\ell_2(\mathcal{B})$, where $\mathcal{B} = \mathbb{Z}^2/\{0\}$; for $k=(k^0,\vec k=\vec 0)$ on the anti-symmetric subspace of $\ell_2(B)$. $\hat K(-\vec\xi\,',-\vec\eta\,',k)$ acts as an energy-dependent non-local potential in the non-relativistic lattice Schrödinger operator analogy.

We now find L, the ladder approximation to K which is given by the κ^2 contribution to K (the lowest non-vanishing order in κ). Writing $D = D_0 + D^T$, where $D^T = D - D_0$ is the connected four-point function, we have the Neumann series $K = D_0^{-1}D^TD_0^{-1} - \sum_{n=2}^\infty(-1)^n(D_0^{-1}D^T)^nD_0^{-1}$ and, expanding in κ, we obtain $D^T(x_1,x_2,x_3,x_4) = -\frac{3}{4}\kappa^2\sum_{j=1,2;\,\epsilon=\pm1}\delta(x_2-x_1+\epsilon e^j)[-\delta(x_1-x_3)\delta(x_2-x_4)+\delta(x_1-x_4)\delta(x_2-x_3)] + \mathcal{O}(\kappa^3)$ and $D_0(x_1,x_2,x_3,x_4) = -\delta(x_1-x_3)\delta(x_2-x_4)+\delta(x_1-x_4)\delta(x_2-x_3)+\mathcal{O}(\kappa^3)$. Thus, $L(x_1,x_2,x_3,x_4) = -\frac{3}{16}\kappa^2\sum_{j=1,2;\,\epsilon=\pm1}\delta(x_2-x_1+\epsilon e^j)[-\delta(x_1-x_3)\delta(x_2-x_4)+\delta(x_1-x_4)\delta(x_2-x_3)]+\mathcal{O}(\kappa^3)$ and, on the space of odd functions, $\hat L(\vec\xi,\vec\eta,k^0) = \dfrac{3}{8}\kappa^2\,\delta(\vec\xi-\vec\eta)\sum_{j=1}^2[\delta(\vec\xi-e^j)+\delta(\vec\xi+e^j)]$, where we use the abbreviated notation k^0 for $k=(k^0,\vec k=\vec 0)$. The $\mathcal{O}(\kappa^2)$ term for D comes from computing the one fermion-anti-fermion pair (but with spin indices that do not allow us to identify it with a quark-anti-quark meson!) exchange contribution between two pairs of baryons separated by one lattice space unit. Details of similar calculations, involving the computation of Fermi averages and gauge group integrals, can be found in [14].

Now we give the representation of D_0 that is to be used in the B-S equation. In relative coordinates, $D_0(\vec\xi,\vec\eta,\tau) = -G(\tau+\vec\xi)G(\tau+\vec\eta)+G(\tau)G(\tau+\vec\xi+\vec\eta)$, and we obtain a spectral representation for $\hat D_0(\vec\xi,\vec\eta,k^0)$ by separating out the $\tau^0=0$ term in $D_0(\vec\xi,\vec\eta,\tau)$, and using the spectral representation for G, for $\tau^0\neq 0$, to get

$$\hat D_0(\vec\xi,\vec\eta,k^0) = -2(2\pi)^{-2}\int_{\mathbf{T}^2} M(\vec p,k^0)\,\sin\vec p.\vec\xi\,\sin\vec p.\vec\eta\,d\vec p\,,$$

where $M(\vec p,k^0) = 2\pi\tilde G(\vec p)^2 + (2\pi)^5\int_{-1}^1\int_{-1}^1 f(k^0,\lambda^0\lambda'^0)\, d_{\lambda^0}\alpha_{\vec p}(\lambda'^0)d_{\lambda^0}\alpha_{\vec p}(\lambda'^0)$.

We now obtain the solution of the B-S equation for $\hat D(\vec\xi,\vec\eta,k^0)$ in the ladder approximation. Substituting $\hat L$ in $\hat D(\vec\xi,\vec\eta,k^0)$ and suppressing the k^0 dependence, we get

$$\hat D(\vec\xi,\vec\eta) = \hat D_0(\vec\xi,\vec\eta) + a\kappa^2\sum\hat D_0(\vec\xi,\epsilon e_\mu)\hat D(\epsilon e_\mu,\vec\eta)\,,$$

with the sum here running over $\mu=1,2$ and $\epsilon=\pm1$. Successively, taking $\vec\xi = \pm e_\mu$, $\mu=1,2$, with $\vec\eta$ fixed, we obtain a system of four linear equations for $\hat D(\epsilon e_\mu,\vec\eta)$. However, using the symmetries $\hat D_0(-\vec\xi,\vec\eta) = -\hat D_0(\vec\xi,\vec\eta)$,

$\hat{D}_0(\vec{\xi},-\vec{\eta}) = -\hat{D}_0(\vec{\xi},\vec{\eta})$ and the same for $\hat{D}(\vec{\xi},\vec{\eta})$, we can reduce the above to a system of two linear equations,

$$\hat{D}(e_\mu,\vec{\eta}) = \hat{D}_0(e_\mu,\vec{\eta}) + 2a\kappa^2 \sum_{\nu=1}^{2} \hat{D}_0(e_\mu,e_\nu)\hat{D}(e_\nu,\vec{\eta}) \,,$$

for $\mu = 1,2$, with the solution $\hat{D}(\vec{\xi},\vec{\eta}) = \hat{D}_0(\vec{\xi},\vec{\eta}) + 2a\kappa^2 \sum_{\mu,\nu=1}^{2} \hat{D}_0(\vec{\xi},e_\mu)\frac{\mathcal{M}_{\mu\nu}}{W}\hat{D}_0(e_\nu,\vec{\eta})$, where $W = \det\mathcal{M}$, and the 2×2 matrix $\mathcal{M}$ has elements $\mathcal{M}_{11} = 1 - 2a\kappa^2\hat{D}_0(e_2,e_2)$, $\mathcal{M}_{12} = -2a\kappa^2\hat{D}_0(e_1,e_2)$, $\mathcal{M}_{21} = -2a\kappa^2\hat{D}_0(e_2,e_1)$ and $\mathcal{M}_{22} = 1 - 2a\kappa^2\hat{D}_0(e_1,e_1)$.

To determine bound states below the two-baryon threshold we recall that they occur as k^0 singularities of $\hat{D}(\vec{\xi},\vec{\eta},k^0)$ which in turn occur as zeroes of $W(k^0) = 1 - 2a\kappa^2[\hat{D}_0(e^1,e^1,k^0) \mp \hat{D}_0(e^1,e^2,k^0)]$. That there are no bound states follows by showing that $W(k^0 = i\chi)$ has no zero for $\chi \in [0,2m_\kappa)$. We note that $[-\hat{D}_0(e^1,e^1,k^0 = i\chi)]$ is finite, positive and monotone increasing in χ as $M(\vec{p},i\chi)$ is, where we use the above measure decomposition for $d_{\lambda^0}\alpha_{\vec{p}}(\lambda^0)$, and the fact that $w(\vec{p}) \geq w(\vec{0}) = m_\kappa$ and the λ^0 support of $d\nu(\lambda^0,\vec{\lambda})$ is contained in $|\lambda^0| \leq |\kappa|^{5-\epsilon}$. The monotonicity of $M(\vec{p},i\chi)$ follows from that of $f(i\chi,\lambda^0\lambda'^0)$. By the Cauchy-Schwarz inequality, $|\hat{D}_0(e^1,e^2,k^0 = i\chi)| \leq -\hat{D}_0(e^1,e^1,k^0 = i\chi)$, and since $a = 3/4 > 0$, $W(\chi) \geq 1$.

Before we close, it is instructive to establish the correspondence of the B-S equation near the threshold and in the diagonal (ladder) approximation with the non-relativistic lattice Schrödinger resolvent equation. For $k^0 = i\chi$, $\chi = 2m_\kappa - \epsilon$, $0 < \epsilon \lll 1$. The product of the one-particle contributions are the dominant ones in D_0, and approximating $Z(\vec{p})$ and $w(\vec{p})$ by their small $|\vec{p}|$ values, D_0 has the approximate expression $-\frac{2}{(2\pi)^2}\int_{\mathbf{T}^2} \frac{1}{2\kappa^3|\vec{p}|^2+\epsilon} \sin\vec{p}.\vec{\xi}\, \sin\vec{p}.\vec{\eta}\;\; d\vec{p}$. Up to the constant, this is minus the lattice Schrödinger free resolvent $-[-2\kappa^3\Delta + \epsilon]^{-1}$, where Δ is the lattice Laplacian operator in $\ell_2(\mathbb{Z}^d)$, restricted to the antisymmetric subspace which is manifested by the sine function factors, and is in contrast with the bosonic case where only cosines appear [13, 18]. In the diagonal approximation, the B-S kernel is given by $\hat{K}_d(\vec{\xi},\vec{\eta},k^0) = k(\vec{\xi})\delta(\vec{\xi}-\vec{\eta})$, where $k(\vec{\xi}) = K(x_1,x_2,x_1,x_2)$, with $x_2 - x_1 = (0,\vec{\xi})$. The B-S equation then becomes $\hat{D}(\vec{\xi},\vec{\eta},k^0) = \hat{D}_0(\vec{\xi},\vec{\eta},k^0) + \int \hat{D}_0(\vec{\xi},\vec{\xi}',k^0)k(\vec{\xi}')\hat{D}(\vec{\xi}',\vec{\eta},k^0)d\vec{\xi}'$, which we identify as the standard one-particle lattice Schrödinger resolvent equation $(H-z)^{-1} = (H_0-z)^{-1} - (H_0-z)^{-1}V(H-z)^{-1}$, with $z = -\epsilon$, free Hamiltonian $H_0 = -2\kappa^3\Delta$, potential $V = k$, and restricted to the anti-symmetric subspace. A positive $k(\vec{\xi})$ means a repulsive potential. A similar calculation involving a lattice Yukawa model shows an attractive potential, as expected. Here, this repulsion excludes bound states. Indeed, as we have seen, $\hat{L}$ is our κ^2 approximation to $\hat{K}$, and $k(\vec{\xi})$ is given by $a\kappa^2\sum_{\mu=1}^{2}[\delta(\vec{\xi}-e^\mu) + \delta(\vec{\xi}+e^\mu)]$, $a = 3/8 > 0$. Whether the absence of a bound state is a limitation of our simplified model or the region of parameters we consider is currently being investigated.

Acknowledgments

Work partially supported by Pronex, CNPq and FAPESP.

[1] R. Machleidt, Nucl. Phys. **A 689**, 11c (2001).

[2] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).

[3] K. Wilson, in *New Phenomena in Subnuclear Physics*, Part A, A. Zichichi ed. (Plenum Press, NY, 1977).

[4] T. Banks et al., Phys. Rev. **D 15**, 1111 (1977).

[5] K. Osterwalder, and E. Seiler, Ann. Phys. (NY) **110**, 440 (1978).

[6] E. Seiler, Lect. Notes in Phys. **159** (1982).

[7] J. Fröhlich, and C. King, Nucl. Phys **B 290**, 157 (1987).

[8] I. Montvay, Rev. Mod. Phys. **59**, 263 (1987).

[9] F. Myhrer, and J. Wroldsen, Rev. Mod. Phys. **60**, 629 (1988).

[10] D. Schreiber, Phys. Rev. **D 48**, 5393 (1993).

[11] M. Creutz, Nucl. Phys. **B** (Proc. Suppl.) **94** 219 (2001).

[12] P.A. Faria da Veiga, M. O'Carroll, and R. Schor, Phys. Rev. **D 67**, 017501 (2003).

[13] R.S. Schor, and M. O'Carroll, Phys. Rev. **E 62**, 1521 (2000). J. Stat. Phys. **99**, 1207 (2000), J. Stat. Phys. **99**, 1265 (2000) and J. Stat. Phys. **109**, 279 (2002).

[14] P.A. Faria da Veiga, M. O'Carroll, and R. Schor, pre-print (2003), available from www.icmc.usp.br/∼veiga/baryons.pdf.

[15] A. Francisco Neto, P.A. Faria da Veiga, and M. O'Carroll, pre-print (2003), available from www.icmc.usp.br/∼veiga/mesons.pdf.

[16] B. Simon *Statistical Mechanics of Lattice Models* (Princeton University Press, Princeton, 1994).

[17] R. Schor, Nucl. Phys. **B 222**, 71 (1983) and Nucl. Phys. **B 231**, 321 (1984).

[18] P.A. Faria da Veiga, and M. O'Carroll, J. Phys. **A**35, 4541 (2002).